In a nutshell: Inverse quadratic interpolation

Given a continuous real-valued function f of a real variable with three initial approximations of a root x_{-2}, x_{-1} and x_0 where $|f(x_{-2})| \geq |f(x_{-1})| \geq |f(x_0)| > 0$, rearranging them if necessary. If one is zero, we have already found a root. This algorithm uses iteration, quadratic interpolation and a solution to the quadratic equation to approximate a root.

Unlike Muller's method which approximates f, here we will approximate f^{-1}, and if x is a root, then $f(x) = 0$; hence locally $f^{-1}(0) = x$.

Parameters:

$\varepsilon_{\text{step}}$	The maximum error in the value of the root cannot exceed this value.
ε_{abs}	The value of the function at the approximation of the root cannot exceed this value.
N	The maximum number of iterations.

1. Let $k \leftarrow 0$.
2. If $k > N$, we have iterated N times, so stop and return signalling a failure to converge.
3. If any two of $f(x_{k-2})$, $f(x_{k-1})$ or $f(x_k)$ are equal return signalling a failure to converge.
4. If $|f(x_{k-2})| > |f(x_{k-1})| > |f(x_k)| > 0$ does not hold, rearrange the values so as to ensure this is true.
5. The next approximation to the root be the quadratic polynomial that interpolates the three points $(f(x_{k-2}), x_{k-2})$, $(f(x_{k-1}), x_{k-1})$ and $(f(x_k), x_k)$ evaluated at zero, so

   let $$x_{k+1} \leftarrow \frac{\left(f(x_k) - f(x_{k-1})\right) f(x_k) f(x_{k-1}) x_{k-2} + \left(f(x_{k-1}) - f(x_{k-2})\right) f(x_{k-1}) f(x_{k-2}) x_k + \left(f(x_{k-2}) - f(x_k)\right) f(x_{k-2}) f(x_k) x_{k-1}}{\left(f(x_k) - f(x_{k-1})\right)\left(f(x_{k-1}) - f(x_{k-2})\right)\left(f(x_{k-2}) - f(x_k)\right)}.$$

   a. If x_{k+1} is not a finite floating-point number, so return signalling a failure to converge.
   b. If $|x_{k+1} - x_k| < \varepsilon_{\text{step}}$ and $|f(x_{k+1})| < \varepsilon_{\text{abs}}$, return x_{k+1}.
6. Increment k and return to Step 2.

Note that we can include a bracketing component this algorithm by always ensuring that two of the three points have opposite signs when f is evaluated at them. Then, when a root is found, a point can always be chosen to ensure that the root is still bracketed.

Convergence

If h is the error, it can be show that the error decreases according to $O(h^{\mu})$ where $\mu \approx 1.8393$ is the real root of $x^3 - x^2 - x - 1 = 0$.